Facets in Argumentation: A Formal Approach to Argument Significance

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Introduction

Abstract argumentation [Dung, 1995; Bench-Capon and Dunne, 2007] is a formalism for modeling and evaluating arguments and its reasoning problems has many applications in artificial intelligence (AI) [Amgoud and Prade, 2009; Rago *et al.*, 2018]. The semantics is based on sets of arguments (called *extensions*) satisfying certain conditions regarding their mutual relationship, such as being stable (every argument is either accepted, or attacked by an accepted argument) or admissible [Dung, 1995]. To compute such extensions of a framework, various practical solvers for decision and reasoning tasks [Egly *et al.*, 2008; Niskanen and Järvisalo, 2020; Thimm *et al.*, 2021; Alviano, 2021] compete biennially in the ICCMA competition [Thimm *et al.*, 2024]. Qualitative reasoning tasks (e.g., extension computation, credulous or skeptical acceptance) are computationally efficient [Dvořák, 2012], but offer limited insight into argument preferences. To address this, enumeration, counting, and quantitative reasoning have been explored and classified [Fichte *et al.*, 2023; Fichte *et al.*, 2024], enabling probabilistic analysis. While enumeration is feasible with few extensions, many semantics produce a large number of them. Nonetheless, users may wish to explore extension spaces further—for example, by restricting or diversifying extensions, identifying resilient arguments, performing sanity checks, evaluating LLM-generated frameworks, or deriving explanations. Assessing the significance of individual arguments within a framework is central. Existing approaches compute significance using quantitative measures over extensions that contain specific arguments or support particular claims. This typically involves counting extensions—an inherently costly task [Fichte *et al.*, 2024]. Example 1 illustrates the challenges of comparing argument significance across an extension space.

Example 1. *Consider the AF F, depicted in Figure 1. Intuitively, a preference for savory flavors excludes choices like maple syrup, just as sweet opposes burrito. Maple syrup or small portions are unexpected at Taco Bell. Burritos, though can be made at home, doing so requires expensive ingredients—avoided by making either small portion or opting for Taco Bell. Stable extensions of F are: $\{W, M, P\}$, $\{S, B, P\}$, $\{S, B, T\}$. Here, it is not immediate to compare the significance of accepting/rejecting certain arguments to each other.*

In this paper, we propose a notion of facets defined as arguments accepted in some extension (credulous) but not by all extensions (skeptical). Facets quantify the uncertainty of arguments in extensions, providing a measure of their indeterminacy within the framework. Furthermore, they can be utilized to evaluate the significance of specific arguments. Example 2 provides a brief intuition.

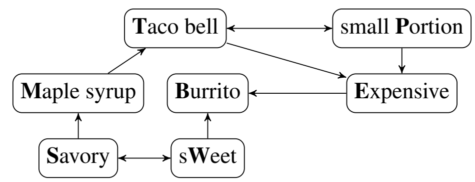

Figure 1: An example argumentation framework.

Example 2. *Reconsider Example 1 where six of the seven arguments are facets under stable semantics, the only exception being E. To compare the relative significance, consider stable extensions of F rejecting the argument "s**W**eet". This includes $\{S, B, P\}$ and $\{S, B, T\}$, thus leaving us with two facets P and T. In contrast,* accepting *"s**W**eet" gives a single stable extension $\{W, M, P\}$ and hence no facets. Accordingly, accepting W eliminates any uncertainty, and we consider accepting "s**W**eet" as more significant than rejecting it.*

Contributions. We introduce facets to abstract argumentation as a reasoning tool for significance and filtering extensions. Furthermore, we present a comprehensive complexity analysis (Table 1) for various qualitative and quantitative problems involving facets. Finally, we present experiments that demonstrate the feasibility of our framework and evaluate our implementation on instances of the ICCMA competition.

Related Work. Facets were initially proposed for answer-set programming (ASP) by Alrabbaa *et al.* [2018] as a tool to navigate large solution spaces and their complexity has been systematically classified [Rusovac *et al.*, 2024]. Note that ASP-navigation is based on forbidding or enforcing atoms in a program using integrity constraints. In contrast, argumentation facets enable approving or disapproving arguments, while not necessarily removing the extensions entirely leading to a natural notion of significance of an argument. The computational complexity of credulous and skeptical reasoning is well studied [Dvořák, 2012]. Here, we ask for the concrete complexity of facets and whether it provides a theoretical benefit

Problems/σ	σ_1	σ_2	σ_3
$\textsc{IsFacet}_\sigma$	**P**	**NP**	$\mathbf{\Sigma_2^P}$
$\textsc{Facets}_\sigma^{\geq k}$	**P**	**NP**	$\mathbf{\Sigma_2^P}$
$\textsc{Facets}_\sigma^{\leq k}$	**P**	co**NP**	$\mathbf{\Pi_2^P}$
$\textsc{Facets}_\sigma^{=k}$	**P**	**DP**	$\in \mathbf{DP}_2$
$\mathbb{S}_\sigma[F,\ell]$	"$\in$ **P**"	"$\in \mathbf{\Delta_2^P}$"	"$\in \mathbf{\Delta_3^P}$"

Table 1: A complexity overview for semantics $\sigma_1 \in \{\mathsf{conf}, \mathsf{naiv}\}$, $\sigma_2 \in \{\mathsf{adm}, \mathsf{stab}, \mathsf{comp}\}$, and $\sigma_3 \in \{\mathsf{pref}, \mathsf{semiSt}, \mathsf{stag}\}$. $\textsc{IsFacet}_\sigma$ asks whether a given argument is a facet; $\textsc{Facets}_\sigma^{\cdot k}$ asks whether there are at least ($\geq k$), at most ($\leq k$), or exactly ($= k$) facets. $\mathbb{S}_\sigma[F,\ell]$ asks what the significance of approving ($\ell = a$) or disapproving ($\ell = \bar{a}$) of a σ-facet a in AF F is. "$\in \mathbf{\Delta_i^P}$" slightly abuses notation meaning that it can be computed by a deterministic polynomial-time Turing machine with access to a $\mathbf{\Sigma_{i-1}^P}$ oracle.

over projected counting and enumerating extensions.

Facet Reasoning

We assuming basic familiarity with mostly studied argumentation semantics, denoted henceforth by σ. Given an AF F, $\sigma(F)$ denotes the set of all σ-extensions of F. Moreover, $\mathsf{C}_\sigma = \bigcup_{E\in\sigma(F)} E$ denotes credulously, and $\mathsf{S}_\sigma = \bigcap_{E\in\sigma(F)} E$ denotes skeptically accepted arguments of F under σ. A σ-facet is an argument which is accepted in some, but not all σ-extensions. Formally, an argument a is a *σ-facet* if $a \in \mathsf{C}_\sigma \setminus \mathsf{S}_\sigma$. Given an AF F and semantics σ, $\mathcal{F}_\sigma(F)$ denotes the set of all σ-facets in F. We consider the following reasoning problems: The problem $\textsc{IsFacet}_\sigma$ asks, given an AF $F = (A, R)$ and an argument $a \in A$, is a a σ-facet in F? The problems $\textsc{Facets}_\sigma^{=k}$, $\textsc{Facets}_\sigma^{\geq k}$ and $\textsc{Facets}_\sigma^{\leq k}$ has an integer k as additional input, and ask whether an input $F = (A, R)$ has exactly, at least, or at most k σ-facets, respectively.

Significance via Facets Our notion of significance adopts a decision-driven perspective. We define significance of arguments in terms of the influence of a decision to eliminate the degree of freedom (on choices of remaining arguments), which actually complements counting/conditional probability. Observe that, counting approaches assess the plausibility of arguments in terms of their likelihood of being accepted (global perspective on all extensions). However, we measure how much the acceptance of an argument decreases freedom (or increases the significance of the decision). Intuitively, a higher significance score indicates that a specific decision does have a huge influence on the remaining facets. Furthermore, the acceptance or rejection of arguments across extensions reflects inherent uncertainty in an AF, with the number of facets indicating its degree. For an argument a denote by $\bar{a}$ the complement/negation of a. E.g., an argument is approved (a) versus not approved ($\bar{a}$). A facet $\ell \in \{a, \bar{a}\}$ can be seen as the *uncertainty* regarding a, since a can either be included in, or excluded from extensions. Let F be an AF and σ be a semantics. For an argument a, let $\sigma^a(F)$ denotes the σ-extensions in F *approving* the argument a defined as $\sigma^a(F) = \{E \in \sigma(F) \mid a \in E\}$. Likewise, $\sigma^{\bar{a}}(F) = \{E \in \sigma(F) \mid a \notin E\}$ represents σ-extensions in F *disapproving* a. Now, let C_σ^a (resp., S_σ^a) be the arguments in some (all) $E \in \sigma^a(F)$ and $\mathcal{F}_\sigma^a(F)$ denotes the σ-facets by considering extensions in $\sigma^a(F)$ (i.e., $\mathsf{C}_\sigma^a \setminus \mathsf{S}_\sigma^a$).

Approving a facet $\ell \in \{a, \bar{a}\}$ reduces the uncertainty regarding the remaining arguments in F by restricting the extensions space to sets (not) containing a. Further, approving ℓ can render a facet argument $b \in A$ non-facet. This holds since, either $b \in E$ for each $E \in \sigma^\ell(F)$ but $b \notin E$ for each $E \in \sigma(F)$, or $b \notin E$ for any $E \in \sigma^\ell(F)$ but $b \in E$ for some $E \in \sigma(F)$. Intuitively, we say that the uncertainty of such an argument b has been *resolved* by approving ℓ.

Let σ be a semantics, $a \in A$ be a σ-facet and $\ell \in \{a, \bar{a}\}$. The observation that "ℓ reduces the uncertainty among remaining arguments" leads to the notion of *significance* of ℓ under semantics σ. For an AF F, we define: $\mathbb{S}_\sigma[F,\ell] := {}^{|\mathcal{F}_\sigma(F)| - |\mathcal{F}_\sigma^\ell(F)|}/_{|\mathcal{F}_\sigma(F)|}$. Intuitively, approving an argument a is less significant if many uncertain arguments (facets) remain in $\mathcal{F}_\sigma^a(F)$. Similarly, disapproving a (and thus approving $\bar{a}$) is less significant if many facets remain in $\mathcal{F}_\sigma^{\bar{a}}(F)$.

Example 3 (Arguments Significance)**.** *Reconsider the AF F from Example 1 with stable extensions* $\mathsf{stab}(F) = \{\{W, M, P\}, \{S, B, P\}, \{S, B, T\}\}$. *While Example 2 gave an intuition of significance, Table 2 presents precise values for each argument. As outlined, the argument W has score* 1*, and is thus more significant than $\bar{W}$ (score* 2/3*).*

$\ell \in$	$\{W, M, T, \bar{S}, \bar{B}, \bar{P}\}$	$\{S, B, \bar{W}, \bar{M}\}$	$\{P, \bar{T}\}$
$\left\lvert\mathcal{F}_{\mathsf{stab}}^\ell(F)\right\rvert$	0	2	4
$\mathbb{S}_{\mathsf{stab}}[F,\ell]$	1	$\frac{2}{3}$	$\frac{1}{3}$

Table 2: Argument significance for the AF from Example 3.

Conclusion.

We defined a new perspective on exploring significance of arguments in extensions of an abstract argumentation framework. We present a comprehensive complexity analysis for deciding whether an argument is a facet ($\textsc{IsFacet}$) and deciding whether an argumentation framework has at least k ($\textsc{Facets}_\sigma^{\geq k}$), at most k ($\textsc{Facets}_\sigma^{\leq k}$), and exactly k facets ($\textsc{Facets}_\sigma^{=k}$). We establish that the complexity ranges between **P** and $\mathbf{DP}_2$, including tight lower bounds for most cases (see Table 1). While our primary focus lies on establishing a comprehensive complexity picture, our implementation allows computing the number of facets practically for concrete abstract argumentation frameworks building on top of existing solvers. Thereby, we demonstrate that we can practically compute facets as well and our new notions are ready to use.

For future work, we plan to investigate techniques whether significance originating from facets can be extended to arguments depending on each other and notions of fairness in argumentation frameworks. Moreover, we aim to explore facets in three-valued variants of complete and preferred semantics [Wu *et al.*, 2010], which distinguish rejected arguments by whether they are undefended or attacked by accepted arguments. From a practical perspective, we believe that it would be interesting to integrate facet-based reasoning and significance computation into modern SAT-based argumentation solvers.

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