Generalized Safe Conditional Syntax Splitting of Belief Bases (Extended Abstract)

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Abstract

Splitting techniques in knowledge representation help focus on relevant parts of a belief base and reduce the complexity of reasoning generally. We propose a generalization of safe conditional syntax splittings that broadens the applicability of splitting postulates for inductive inference from conditional belief bases. In contrast to safe conditional syntax splitting, our generalized notion supports syntax splittings of a belief base Δ where the subbases of Δ may share atoms and nontrivial conditionals.

1 Introduction

For epistemic reasoning, both from a cognitive point of view and from the point of view of effective implementations, it is often vital to focus on the relevant parts, and leave aside facts and knowledge irrelevant for the question at hand, thus enabling local reasoning [Pearl, 1988]. This is the basic motivation underlying the concept of syntax splitting [Parikh, 1999; Peppas *et al.*, 2015; Kern-Isberner and Brewka, 2017], and of the related idea of minimum irrelevance [Weydert, 1998]. Under the motto "syntax splitting = relevance + independence", respecting syntax splitting was formalized for inductive inference from conditional belief bases [Kern-Isberner *et al.*, 2020], taking splittings over a belief base Δ into account where the subbases Δ_1, Δ_2 are given over disjoint subsignatures of Δ. This condition is a severe restriction in practice because full disjointness is often not the case. The concept of conditional syntax splitting [Heyninck *et al.*, 2023] is an approach to overcome this restriction by allowing Δ_1 and Δ_2 to overlap syntactically. A safety condition ensures that semantic (conditional) independence holds given the joint atoms, enabling local reasoning within the subbases. It has been shown that the postulate of conditional independence (CInd) for safe conditional splittings precisely characterizes avoiding the drowning effect [Pearl, 1990; Benferhat *et al.*, 1993], yielding the first formal definition of the notorious drowning problem that had been described before only by specific examples [Heyninck *et al.*, 2023]. However, it has been shown recently that the safety condition given in [Heyninck *et al.*, 2023] requires that every conditional in the intersection of Δ_1 and Δ_2 is a trivial self-fulfilling conditional, that cannot be falsified [Beierle *et al.*, 2024]. For avoiding this restriction and broadening appliction possibilites of syntax splitting, we develop a generalization of safe conditional syntax splittings. This generalization allows us to greatly increase both the amount of splittings and the amount of belief bases where splittings can be exploited for inductive reasoning. We characterize the splittings useful for inductive inference by identifying the subclass of genuine splittings, separating them from the class of simple splittings that have no benefits for inductive inference. This is an extended abstract of the paper accepted for IJCAI2025 [Spiegel *et al.*, 2025].

2 Formal Basics

We consider a finitely generated propositional language $\mathcal{L}$ over a signature Σ with atoms $a, b, c, \dots$ and with formulas $A, B, C, \dots$ As models of formulas we will use the set Ω of *possible worlds* over $\mathcal{L}$. We will use ω both for the model and the corresponding conjunction of all positive or negated atoms. For subsets Σ_i of Σ, let $\mathcal{L}(\Sigma_i)$ denote the propositional language defined by Σ_i, with associated set of interpretations $\Omega(\Sigma_i)$. With ω^i we denote the reduct of ω to Σ_i [Delgrande, 2017]. Conditionals $(B|A)$ are meant to express plausible, yet defeasible rules "If A then plausibly B". $(B|A)$ is *verified* by ω if $\omega \models AB$ and *falsified* by ω if $\omega \models A\overline{B}$. A conditional $(B|A)$ is called *self-fulfilling*, or *trivial*, if $A \models B$, i.e., there is no world that falsifies it. A belief base Δ is a finite set of conditionals, and we focus on (strongly) consistent belief bases in the sense of [Pearl, 1990; Goldszmidt and Pearl, 1996]. A semantic framework for interpreting conditionals are *ordinal conditional functions (OCFs)* $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$ with $\kappa^{-1}(0) \neq \emptyset$, also called *ranking functions* [Spohn, 1988]. Intuitively, less plausible worlds are assigned higher numbers. Formulas are assigned the rank of their most plausible models, i.e. $\kappa(A) := \min\{\kappa(\omega) \mid \omega \models A\}$. A conditional $(B|A)$ is *accepted* by κ written as $\kappa \models (B|A)$, iff $\kappa(AB) < \kappa(A\overline{B})$. A belief base Δ is accepted by κ written $\kappa \models \Delta$ iff κ accepts all its conditionals.

The nonmonotonic inference relation $\mid\!\sim_\kappa$ induced by κ is

$$A \mid\!\sim_\kappa B \quad \text{iff} \quad A \equiv \bot \text{ or } \kappa(AB) < \kappa(A\overline{B}). \tag{1}$$

To formalize inductive inference from belief bases, [Kern-Isberner *et al.*, 2020] introduced the notion of an *inductive inference operator* which is a mapping $\mathbf{C}$ that assigns to each belief base $\Delta \subseteq (\mathcal{L} \mid \mathcal{L})$ an inference relation $\mid\!\sim_\Delta$ on $\mathcal{L}$, i.e., $\mathbf{C} : \Delta \mapsto \mid\!\sim_\Delta$, such that Direct Inference (DI): If $(B|A) \in \Delta$ then $A \mid\!\sim_\Delta B$, and Trivial Vacuity (TV): $A \mid\!\sim_\emptyset B$ implies $A \models B$ are satisfied.

3 Generalized Safe Conditional Syntax Splitting

We introduce the notion of *generalized safe* conditional syntax splitting as a generalization of safe conditional syntax splitting [Heyninck *et al.*, 2023] that allows us to greatly increase the number of splittings exploitable for inductive inference, and to also increases the amount of belief bases that admit such a splitting at all.

A belief base Δ can be *split into subbases* Δ_1,Δ_2 *conditional on a subsignature* Σ_3, if there are $\Sigma_1, \Sigma_2 \subseteq \Sigma$ such that $\Delta_i = \Delta \cap (\mathcal{L}(\Sigma_i \cup \Sigma_3) \mid \mathcal{L}(\Sigma_i \cup \Sigma_3))$ for $i \in \{1, 2\}$ and $\{\Sigma_1, \Sigma_2, \Sigma_3\}$ is a partition of Σ. This is denoted as

$$\Delta = \Delta_1 \bigcup\nolimits_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3. \tag{2}$$

We denote the intersection of Δ_1 and Δ_2 with $\Delta_3 = \Delta_1 \cap \Delta_2$.

Definition 1. *A belief base* $\Delta = \Delta_1 \bigcup_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3$ *can be* generalized safely split into subbases Δ_1, Δ_2 conditional on a subsignature Σ_3, *writing*

$$\Delta = \Delta_1 \bigcup\nolimits^{\mathsf{gs}}_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3 \tag{3}$$

if the following generalized safety property *holds for* $i, i' \in \{1, 2\}, i \neq i'$*:*

$$\begin{aligned} &\text{for every } \omega^i\omega^3 \in \Omega(\Sigma_i \cup \Sigma_3), \text{ there is } \omega^{i'} \in \Omega(\Sigma_{i'}) \\ &\text{s.t. } \omega^i\omega^3\omega^{i'} \not\models \bigvee\nolimits_{(F|E)\in\Delta_{i'}\setminus\Delta_3} E \wedge \neg F. \end{aligned} \tag{4}$$

The generalized safety condition demands, in essence, that no complete conjunction over Σ_3 may force the falsification of a conditional in $\Delta \setminus \Delta_3$ when considering Σ as a whole. The deciding difference between safe splittings from [Heyninck *et al.*, 2023] and our generalization is that the original safety property demands that no conditional in Δ as a whole can be falsified by a complete conjunction over Σ_3. This begets the side effect that Δ_3 contains only self-fulfilling conditionals, i.e., Δ_1 and Δ_2 may not share "meaningful" conditionals [Beierle *et al.*, 2024] which our generalization avoids.

For inductive inference operators we introduce variants of the postulates (CRel) and (CInd) from [Heyninck *et al.*, 2023] by adapting them to our generalized notion of safety.

Definition 2 (adapted from [Heyninck *et al.*, 2023])**.** *For any* $\Delta = \Delta_1 \bigcup^{\mathsf{gs}}_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3$*, for* $i, i' \in \{1, 2\}$ *with* $i \neq i'$ *and any* $A, B \in \mathcal{L}(\Sigma_i)$*,* $D \in \mathcal{L}(\Sigma_{i'})$ *and a complete conjunction* $E \in \mathcal{L}(\Sigma_3)$*, such that* $DE \not\mid\!\sim_\Delta \bot$*, an inductive inference operator* $\mathbf{C}$ *satisfies*

(CRel$^{\mathrm{g}}$) *if* $AE \mid\!\sim_\Delta B$ *iff* $AE \mid\!\sim_{\Delta_i} B$.

(CInd$^{\mathrm{g}}$) *if* $AE \mid\!\sim_\Delta B$ *iff* $AED \mid\!\sim_\Delta B$.

(CSynSplit$^{\mathrm{g}}$) *if it satisfies (CRel^{g}) and (CInd^{g}).*

For governing inductive inference, we are therefore interested in (generalized) safe conditional syntax splittings. However there exists safe conditional syntax splittings where Δ_1 is a subset of Δ_2 or vice versa. In these cases the postulates (CInd$^{\mathrm{g}}$) and (CRel$^{\mathrm{g}}$) loose their meaning. We identify splittings that are meaningful with respect to inductive inference as so-called *genuine splittings*.

Definition 3. *Let* Δ *be a belief base over a signature* Σ*. A conditional syntax splitting* $\Delta = \Delta_1 \bigcup_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3$ *of* Δ *is called* genuine*, if* $\Delta_1 \not\subseteq \Delta_2$ *and* $\Delta_2 \not\subseteq \Delta_1$

We illustrate the importance of identifying genuine splittings and our generalization of the safety property.

Example 4 (Δ^{rain})**.** *We consider the belief base* $\Delta^{rain} = \{(\overline{s}|r), (\overline{r}|s), (b|sr), (o|s\overline{r}), (\overline{o}|r), (u|or)\}$*. One of the possible conditionals syntax splittings of* Δ^{rain} *is*

$$\Delta^{rain} = \{(\overline{s}|r), (\overline{r}|s), (b|sr)\} \bigcup^{\mathsf{gs}}_{\{b\},\{o,u\}} \{(\overline{s}|r), (\overline{r}|s), (o|s\overline{r}), (\overline{o}|r), (u|or)\} \mid \{s, r\} \tag{5}$$

which, however, is not safe. But the splitting (5) *is both generalized safe and genuine.* Δ^{rain} *has a total of 37 conditional syntax splittings, out of which 32 are generalized safe splittings, but only 16 are safe splittings. Only 5 of the 37 splittings are genuine. For this belief base all genuine splittings are generalized safe, while no safe splitting is genuine.*

4 Inductive Inference Respecting Generalized Safe Conditional Syntax Splitting

A c-representation is a special kind of ranking function, that assigns penalty points to worlds based on the conditionals they falsify [Kern-Isberner, 2001; Kern-Isberner, 2004]. c-Inference [Beierle *et al.*, 2018; Beierle *et al.*, 2021] is the inductive inference $\mid\!\sim^{c\text{-}sk}_\Delta$ obtained by taking all c-representations into account, i.e., $A \mid\!\sim^{c\text{-}sk}_\Delta B$ iff $A \mid\!\sim_\kappa B$ for all c-representations κ of Δ. We show:

Proposition 5. *c-Inference satisfies (CRel^{g}) and (CInd^{g}) and thus (CSynSplit^{g}).*

A selection strategy σ assigns to each belief base a unique c-representation [Beierle and Kern-Isberner, 2021], thus yielding an inductive inference operator via $\mathbf{C}^{c\text{-}rep}_\sigma : \Delta \mapsto \kappa_{\sigma(\Delta)}$ where $\mid\!\sim_{\kappa_{\sigma(\Delta)}}$ is obtained via Equation (1). In general $\mathbf{C}^{c\text{-}rep}_\sigma$ does not satisfy (CSynSplit$^{\mathrm{g}}$). But we introduce the postulate (IP-CSP$^{\mathrm{g}}$) for selection strategies, demanding that, for any $\Delta = \Delta_1 \bigcup^{\mathsf{gs}}_{\Sigma_1,\Sigma_2} \Delta_2 \mid \Sigma_3$, the selection strategy σ assigns the same impact to the conditionals of Δ_i in Δ as in Δ_i, i.e., $\sigma(\Delta_i) = \sigma(\Delta)|_{\Delta_i}$. We show:

Proposition 6. *Let* σ *be a selection strategy that satisfies (IP-CSP^{g}). Then* $\mathbf{C}^{c\text{-}rep}_\sigma$ *satisfies (CRel^{g}) and (CInd^{g}) and thus (CSynSplit^{g}).*

Also system W [Komo and Beierle, 2020; Komo and Beierle, 2022] satisfies (CSynSplit$^{\mathrm{g}}$), while system Z [Pearl, 1990] only satisfies (CRel$^{\mathrm{g}}$), but not (CInd$^{\mathrm{g}}$) and thus not (CSynSplit$^{\mathrm{g}}$).

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