On Temporal ASP with Eager Unfoldable Operators
(Extended Abstract)

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Abstract

Temporal Equilibrium Logic (TEL) extends Answer Set Programming (ASP) with linear-time temporal operators (LTL), enabling reasoning about dynamic systems. However, TEL enforces strong minimization criteria that may preclude intuitive models. Liveness formulas, for instance, tend to fail to have infinite equilibrium models, as TEL minimization postpones satisfaction forever. We address this limitation by introducing eager temporal operators (eager Until, eager Release, etc.), and present non-disjunctive temporal programs (NDTP) as a framework for modeling dependencies, inertia, and non-determinism. Deciding satisfiability of NDTPs is possible in exponential time and in fact PSPACE-complete.

Answer Set Programming (ASP) [Brewka *et al.*, 2011; Lifschitz, 2019] has been widely applied in dynamic environments, including applications in planning, multi-agent systems, and reasoning about evolving domains [Falkner *et al.*, 2018]. ASP semantics is particularly well-suited for such settings due to its capability of expressing transitive closure, addressing the frame problem (via inertia rules), and providing both default and strong negation, which enables rich and flexible modeling possibilities. Temporal Equilibrium Logic (TEL) [Aguado *et al.*, 2023] builds on this foundation by bridging the gap between the expressive power of linear-time temporal logic (LTL) [Pnueli, 1977] operators and the stable semantics of ASP. By integrating these paradigms, TEL provides a unified framework for reasoning about dynamic systems while preserving the non-monotonic reasoning capabilities and desirable properties of ASP.

Our main contributions, described in detail and supplemented with examples and discussion in [Eiter and Soldà, 2025], are briefly summarized as follows.

- We introduce new versions of the temporal operators that unfold eagerly. While they are indifferent in LTL, they suspend minimization and thus allow for stable models in scenarios where such models are intuitively expected. Notably, the new versions are expressible in TEL and the resulting extension of TEL has the same complexity, i.e., TEL-satisfiability is EXPSPACE-complete [Bozzelli and Pearce, 2015].
- We define $NDTP$ as a rule-based fragment of the new language that, on the one hand, overcomes the problem of strong minimization and, on the other hand, has considerably lower complexity. The former is achieved by using eager temporal operators in rule heads and the latter by disallowing the disjunction, which quickly leads to EXPSPACE-hardness [Šimkus, 2010]. We show that $NDTP$ programs have benign properties, e.g. deciding TEL-satisfiability is in feasible in exponential time and model checking in polynomial time.
- Based on $NDTP$, we define a tight version of temporal programs (TTP). By generalizing results for ordinary tight logic programs [Erdem and Lifschitz, 2003], we show that the stable models of TTP programs are obtained as the LTL-models of their temporal Clark's completion that we define. As a result, we obtain a polynomial encoding of TTP into LTL. Furthermore, we show that tightness of temporal programs can be decided efficiently in nlogspace.

In this extended abstract, we concentrate on TEL-satisfiability, for which PSPACE-hardness is a lower bound by well-known results on temporal logic programs, cf. also [Šimkus, 2010]. A PSPACE upper bound was considered plausible in [Eiter and Soldà, 2025]. This in fact turns out to apply, closing the gap and establishing PSPACE-completeness. $NDTP$ is thus a rich modeling language for dynamic domains of the same complexity as classical LTL. Note that this complexity results can be applied also to the deontic extension of TEL defined in [Soldà *et al.*, 2025].

1 Preliminaries

Both TEL and THT [Aguado *et al.*, 2023] share the same syntax as LTL. Here we introduce the grammar

$$F ::= \top \mid \bot \mid p \mid F \vartriangle F \mid \circ F \mid \bullet F \mid F \bigcirc F \qquad (1)$$

where $p \in \mathcal{P}$ for a finite set $\mathcal{P}$ of propositional atoms, $\vartriangle \in \{\wedge, \vee, \rightarrow\}$, and $\bigcirc \in \{\mathbb{U}, \mathbb{R}, \mathbb{S}, \mathbb{T}\}$. Negation is defined as $\neg\phi := \phi \rightarrow \bot$. As usual, $\Box$ (globally) is defined by $\Box\phi := \bot \,\mathbb{R}\, \phi$; $\Diamond$ (eventually) by $\Diamond\phi := \top \,\mathbb{U}\, \phi$; $\blacksquare$ (historically) by $\blacksquare\phi := \bot \,\mathbb{T}\, \phi$; $\blacklozenge$ (once) by $\blacklozenge\phi := \top \,\mathbb{S}\, \phi$; $\hat{\circ}$ (weak next) by $\hat{\circ}\phi := \circ\phi \vee \neg\circ\top$; and $\hat{\bullet}$ (weak previous) by $\hat{\bullet}\phi := \bullet\phi \vee \neg\bullet\top$. For any unary operator u, we let $\times^0\phi$ denote ϕ and $\times^{i+1}\phi$ denote $\mathsf{u}\mathsf{u}^i\phi$, for $i \geq 0$.

The semantics of THT is defined via THT-traces (simply traces, if unambiguous), which are finite or infinite sequences $\langle \mathbf{H}, \mathbf{T} \rangle$ of pairs $\langle H_i, T_i \rangle$, where $H_i \subseteq T_i \subseteq \mathcal{P}$ for each $0 \leq i <$

λ, where λ can be either in $\mathbb{N}$ or ω. Both $\mathbf{H}$ and $\mathbf{T}$ are traces as usual (LTL-traces), i.e., sequences $\mathbf{H} = H_0, H_1, \ldots$ resp. $\mathbf{T} = T_0, T_1, \ldots$ of sets of atoms. Given a THT-trace $\mathbf{I}$ (or an LTL-trace $\mathbf{T}$), we denote its length by $\lambda_{\mathbf{I}}$ (resp. $\lambda_{\mathbf{T}}$).

Definition 1 (THT-Satisfaction)**.** *Satisfaction of a THT formula by a THT-trace $\mathbf{I} = \langle \mathbf{H}, \mathbf{T} \rangle$ at time k, where $0 \leq k$ is integer, is inductively defined as follows:*

1. $\mathbf{I}, k \not\models \bot$ *and* $\mathbf{I}, k \not\models \top$
2. $\mathbf{I}, k \models p$ *if* $p \in H_k$*, for any atom* $p \in \mathcal{P}$
3. $\mathbf{I}, k \models \phi \vee \psi$ *if* $\mathbf{I}, k \models \phi$ *or* $\mathbf{I}, k \models \psi$
4. $\mathbf{I}, k \models \phi \wedge \psi$ *if* $\mathbf{I}, k \models \phi$ *and* $\mathbf{I}, k \models \psi$
5. $\mathbf{I}, k \models \phi \rightarrow \psi$ *if* $\begin{cases} \langle \mathbf{T}, \mathbf{T} \rangle, k \not\models \phi \text{ or } \langle \mathbf{T}, \mathbf{T} \rangle, k \models \psi, \text{ and} \\ \mathbf{I}, k \not\models \phi \text{ or } \mathbf{I}, k \models \psi \end{cases}$
6. $\mathbf{I}, k \models \circ\, \phi$ *if* $k + 1 < \lambda$ *and* $\mathbf{I}, k + 1 \models \phi$
7. $\mathbf{I}, k \models \phi \,\mathbb{U}\, \psi$ *if there is* $j \geq k$ *s.t.* $\mathbf{I}, j \models \psi$, *and for all* $j' \in [k, j)$, $\mathbf{I}, j' \models \phi$
8. $\mathbf{I}, k \models \phi \,\mathbb{R}\, \psi$ *if for all* $j \geq k$ *s.t.* $\mathbf{I}, j \not\models \psi$, *there exists* $j' \in [k, j)$, $\mathbf{I}, j' \models \phi$
9. $\mathbf{I}, k \models \bullet\, \phi$ *if* $\mathbf{I}, k - 1 \models \phi$ *and* $k > 0$
10. $\mathbf{I}, k \models \phi \,\mathbb{S}\, \psi$ *if there is* $j \leq k$ *s.t.* $\mathbf{I}, j \models \psi$, *and for all* $j' \in (j, k)$, $\mathbf{I}, j' \models \phi$
11. $\mathbf{I}, k \models \phi \,\mathbb{T}\, \psi$ *if for all* $j \geq k$ *s.t.* $\mathbf{I}, j \not\models \psi$, *there exists* $j' \in [k, j)$, $\mathbf{I}, j' \models \phi$,

TEL semantics is now as follows [Aguado *et al.*, 2023].

Definition 2. *A trace* $\mathbf{T}$ *is a stable (equilibrium, TEL) model of formula* ϕ *if* (i) $\mathbf{T} \models \phi$*, i.e.,* $\mathbf{T}$ *is an LTL model of* ϕ*, and* (ii) *no* $\mathbf{H} \neq \mathbf{T}$ *exists s.t.* $\langle \mathbf{H}, \mathbf{T} \rangle \models \phi$.

2 PSPACE-membership for $NDTP$

We propose eager variants of $\mathbb{U}_e$, $\mathbb{R}_e$, $\mathbb{S}_e$, and $\mathbb{T}_e$, which unfold deterministically once a $\mathbf{T}$-trace is fixed.

Definition 3. *The eager variant* O_e *of the operator* $\mathsf{O} \in \{\mathbb{U}, \mathbb{R}, \mathbb{S}, \mathbb{T}\}$ *is as follows. For any THT-trace* $\mathbf{I} = \langle \mathbf{H}, \mathbf{T} \rangle$ *and time point* $k \geq 0$,

12. $\mathbf{I}, k \models \phi \,\mathbb{U}_e\, \psi$ *if there exists some* $j \geq k$ *s.t.* $\mathbf{I}, j \models \psi$, *and for all* $j' \in [k, j)$, $\mathbf{I}, j' \models \phi$ *and* $\mathbf{T}, j' \not\models \psi$;
13. $\mathbf{I}, k \models \phi \,\mathbb{R}_e\, \psi$ *if for all* $j \geq k$ *s.t. (a)* $\mathbf{I}, j \not\models \psi$ *or (b)* $\mathbf{T}, j \models \phi$ *and*$\mathbf{I}, j \not\models \phi$*, some* $j' \in [k, j)$ *exists s.t.* $\mathbf{I}, j' \models \phi$;
14. $\mathbf{I}, k \models \phi \,\mathbb{S}_e\, \psi$ *if there is some* $j \leq k$ *s.t.* $\mathbf{I}, j \models \psi$, *and for all* $j' \in (j, k)$, $\mathbf{I}, j' \models \phi$ *and* $\mathbf{T}, j' \not\models \psi$;
15. $\mathbf{I}, k \models \phi \,\mathbb{T}_e\, \psi$ *if for all* $j \geq k$ *s.t. (a)* $\mathbf{I}, j \not\models \psi$ *or (b)* $\mathbf{T}, j \models \psi$ *and* $\mathbf{I}, j \not\models \psi$*, some* $j' \in [k, j)$ *exists s.t.* $\mathbf{I}, j' \models \phi$.

As usual, we can derive further operators such as: $\Box_e$ (eager globally) by $\Box_e \phi := \bot \,\mathbb{R}_e\, \phi$; $\Diamond_e$ (eager eventually) by $\Diamond_e \phi := \top \,\mathbb{U}_e\, \phi$; $\blacksquare_e$ (eager historically) by $\blacksquare_e \phi := \bot \,\mathbb{T}_e\, \phi$; etc.

We introduce $NDTP$s, nondisjunctive temporal programs, a rule-based fragment extending the class considered in [Erdem and Lifschitz, 2003] to the temporal case by incorporating TEL modalities in the body and allowing arbitrary nesting of eager unfoldable operators in the head.

Definition 4. *A $NDTP$ program* π *consists of sets*
(i) $init(\pi)$ *of initial rules of the form* $r : \psi \rightarrow \phi$*, where* ϕ *is either* $\bot$ *or a* head formula *from the grammar*

$$
\begin{aligned}
\phi &::= \eta[\phi] \mid \phi \,\mathsf{O}_e\, \phi \text{ for } \mathsf{O}_e \in \{\mathbb{U}_e, \mathbb{R}_e, \mathbb{S}_e, \mathbb{T}_e\} \\
\psi &::= \psi_1 \mid \psi_2 \\
\psi_1 &::= \eta[\psi_1] \mid \psi_1 \vee \psi_1 \mid \psi_1 \,\mathsf{O}\, \psi_1 \text{ for } \mathsf{O} \in \{\mathbb{U}, \mathbb{R}, \mathbb{S}, \mathbb{T}\} \\
\psi_2 &::= \eta[\psi_2] \mid \neg\gamma \mid \psi_2 \vee \psi_2 \\
\eta[\mu] &::= \top \mid p \mid \circ\eta[\mu] \mid \hat{\circ}\eta[\mu] \mid \bullet\eta[\mu] \mid \hat{\bullet}\eta[\mu] \mid \\
&\quad\;\; \eta[\mu] \wedge \eta[\mu] \mid \mu
\end{aligned}
$$

where $p \in \mathcal{P}$ *and* γ *is an arbitrary TEL formula,* $\eta[\mu]$ *is parameterized by a grammar symbol, and*
(ii) $dyn(\pi)$ *of dynamic rules* $\Box r$*, where* r *is an initial rule.*

The $NDTP$ fragment disallows: (i) nesting of implications outside the scope of a negation, (ii) negation within temporal unfoldable operators, and (iii) disjunctions in the head. It permits (i) eager operators in the head, and (ii) arbitrary formulas under negation. As for deciding TEL-satisfiability of $NDTP$ programs, we have the following result.

Theorem 1. *Deciding whether an $NDTP$ program* π *is TEL-satisfiable is in PSPACE.*

Proof. (Sketch) The proof proceeds by encoding the program π into a Büchi automaton using well-established LTL-to-automata techniques by adapting the *on-the-fly* automata construction for LTL as discussed in [Vardi, 2005] and adding a condition to guarantee the absence of loops.

The LTL-automata construction we build on our proof is the synchronous product of (1) the *local*-automaton and (2) the *fulfillment*-automaton. More in detail, (1) enforces the local consistency of the evaluation of an LTL-formula along the paths and the handling of the unfolding of the temporal operators; (2) enforces that, whenever an $\alpha\mathbb{U}\beta$-formula is regarded to be true in a state, the fulfillment condition (realization of β) will eventually happen and not infinitely postponed.

We can guarantee the absence of self-supported sets of atoms by guessing the truth-value of the sub-formulas at each node and disregarding the nodes where the guess does not match the set of formulas obtainable via iterative immediate consequence until a fixed-point is reached. To derive an atom p, we may have to make assumptions on future states, e.g. if p at the i-th node can be derived by $\circ q \rightarrow p$; in this case, we employ a labeling schema and label q in the next with index i, and, recursively, each future-assumption with the minimal index among the states where it is supporting an atom. We further require for acceptance of a path in the Büchi automaton that for each such index, some state exists where such an index does not appear. The latter ensures that foundedness in the derivation of atoms along the trace is eventually met. □

The argument can be adapted to finite-traces by requiring no propagation of indexes at the last state. Notably, the stronger condition that some state has no index-label is not made. This is because all future assumptions might eventually be justified, but through different patterns for different subformulas. A simple case of such a case is

$$\Box((p_r \mathbb{U}_e q_r) \wedge (p_s \mathbb{U}_e q_s) \wedge (\circ q_r \rightarrow r) \wedge (\circ q_s \rightarrow s))$$

which admits a temporal equilibrium trace where s, p_s and q_r hold at even states, and r, p_r and q_s hold at odd states.

Ethical Statement

There are no ethical issues.

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