On Independence and SCC-Recursiveness in Assumption-Based Argumentation (Extended Abstract)

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1 Introducing Independence to ABA

Determining conditional (in)dependence between variables is an important concern in AI. The ability to recognize and manage (in)dependence is crucial in symbolic reasoning at large [Darwiche and Pearl, 1994; Darwiche, 1997; Lang *et al.*, 2002]. In the remainder of this extended abstract we give an overview on our work [Blümel *et al.*, 2025], where we study conditional independence in Assumption-based Argumentation.

Assumption-based argumentation (ABA) [Bondarenko *et al.*, 1997; Čyras *et al.*, 2018] is a well-known form of computational argumentation, whose building blocks are assumptions (defeasible elements) and inference rules. ABA is widely applicable, e.g., in healthcare [Cyras *et al.*, 2021b], to provide explanations [Cyras *et al.*, 2021a], for causal discovery [Russo *et al.*, 2024], and planning [Fan, 2018]. In all these settings, a good understanding of independence between various components in ABA frameworks (ABAFs) is crucial. We illustrate the main idea of our approach with the following example.

Example 1.1. *Alice, Bob, and Carol plan a tandem trip; we consider assumptions a (*Alice cycles*), b (*Bob cycles*), and c (*Carol cycles*) for each of our protagonists. Naturally, only two of them cycle at the same time, that is, not all our assumptions can be true at once. We capture the relations between our assumptions with inference rules; e. g., "if Bob and Carol cycle then Alice does not" is captured by the rule ($\overline{a} \leftarrow b, c$), here, $\overline{a}$ is the contrary of a. Crucially, if the* weather is bad *(d) nobody can cycle. On the bright side, then there is no need to* water the plants *(w) outside. Also, Alice thinks about* bringing her book *(k), but if she cycles, it might be too heavy.*

In situations where certain assumptions cannot be true at the same time, ABA is a suitable reasoning tool. Based on a deductive system $(\mathcal{L}, \mathcal{R})$, where $\mathcal{L}$ is a set of sentences and $\mathcal{R}$ is a set of inference rules, an ABA framework (ABAF) is a tuple $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$, where $\mathcal{A}$ is a set of assumptions we make and from which we infer according to the rules $\mathcal{R}$, and $^- : \mathcal{A} \rightarrow \mathcal{L}$ a contrary function, which allows us to derive attacks on assumptions.

Example 1.2. *We formalise Example 1.1 as ABAF D with assumptions $\{a, b, c, d, w, k\}$, their contraries, and inference rules, ($\overline{v} \leftarrow d$), ($\overline{d} \leftarrow v$) for $v \in \{a, b, c\}$, and*

$$\overline{a} \leftarrow b, c \quad \overline{b} \leftarrow a, c \quad \overline{c} \leftarrow a, c \quad \overline{k} \leftarrow a \quad \overline{w} \leftarrow d$$

In order to conduct defeasible reasoning with an ABAF $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$ we consider *arguments* that can be built by applying rules to assumptions. More precisely, we consider the fact, that some $p \in \mathcal{L}$ can be derived from a set of assumptions $A \subseteq \mathcal{A}$ according to the rules $\mathcal{R}$ an *(ABA) argument*, and denote this by $A \vdash p$. Furthermore, if p is the contrary $\overline{b}$ of some assumption $b \in \mathcal{A}$ we say the set A *attacks* any set $B \subseteq \mathcal{A}$ with $b \in B$, thus introducing attacks between sets of assumptions. A set of assumptions A is conflict-free if it does not attack itself; admissible if it is conflict-free and defends itself, i.e., attacks all of its attackers. On top of this attack structure, semantics for ABAFs can be defined using labellings, which assign one of the three labels $\texttt{in}, \texttt{out}$, or $\texttt{und}$ to each assumption. A *labelling-based semantics* σ assigns to each ABAF D a set of σ-labellings $\Lambda_\sigma(D)$.

For our example, we recall the labelling-based version of preferred semantics [Schulz and Toni, 2017]. A labelling on some ABAF D is preferred, if sets of in-labeled assumptions only attack out-labeled assumptions(conflict-free), every set attacking an in-labeled assumption contains an out-labeled assumption (defense) and the set of in-labeled assumptions is maximal among all labellings satisfying these two conditions (maximality).

Example 1.3. *The ABAF D from Example 1.2 has four preferred labellings.*

pr-lab	*a*	*b*	*c*	*d*	*w*	*k*
λ_1	*in*	*in*	*out*	*out*	*in*	*out*
λ_2	*in*	*out*	*in*	*out*	*in*	*out*
λ_3	*out*	*in*	*in*	*out*	*in*	*in*
λ_4	*out*	*out*	*out*	*in*	*out*	*in*

Having defined the setting, we are now ready to formulate the topic of our investigation: *under what conditions are two (sets of) assumptions independent from each other, relative to a (potentially empty) set of assumptions that is considered prior knowledge?*

Motivated by the work [Rienstra *et al.*, 2020] on conditional independence in abstract argumentation, we identify (in)dependencies between sets of assumptions in ABA through the compatibility of partial labellings on them. For $A \subseteq \mathcal{A}$, we let $\lambda|_A : A \rightarrow \{\texttt{in}, \texttt{out}, \texttt{und}\}$ denote the restriction of some labelling λ to a partial labelling on A.

Definition 1.4. *Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-})$ be an ABAF, σ be a semantics, and let $A, B, C \subseteq \mathcal{A}$ be disjoint sets of assumptions. Then A* is σ-independent *of* B, given C in D, *written $A \perp_\sigma B \mid C$ iff, for all labellings $\lambda_1, \lambda_2 \in \Lambda_\sigma(D)$, it holds that if $\lambda_1|_C = \lambda_2|_C$ then there is some labelling $\lambda_3 \in \Lambda_\sigma(D)$ s.t. $\lambda_3|_A = \lambda_1|_A$, $\lambda_3|_B = \lambda_2|_B$, and $\lambda_3|_C = \lambda_1|_C = \lambda_2|_C$.*

Intuitively, a set of assumptions A is dependent on another set B under a semantics σ, if knowing the labelling on B excludes some possible σ-labellings on A. Indeed, this labelling-based independence notion allows us to derive the expected (in)dependencies for our Example.

Example 1.5. *Let us find out if Alice and Bob influence each other's cycling activities: knowing that Alice cycles does not give us any information about Bob, Alice could cycle with either Bob or Carol; similarly, if Alice does not cycle we do not know whether Bob cycles; either Bob and Carol cycle together or the weather could have ruined their trip. In contrast, a and b depend on each other, given c: knowing that Carol cycles implies that one of the other two cycles as well, and knowing she does not cycle implies either both or none of the others cycle. And indeed we can infer:*

- *The assumptions a and b are σ-independent wrt. $\emptyset$ for $\sigma = pr$. For this, we identify the labels which are individually assigned to a and b under σ, i.e., a and b can both be labeled* `out` *at the same time (λ_4).*
- *When conditioning on $\{c\}$, the assumptions a and b are dependent under pr semantics. If $\lambda(c) =$* `in`*, then a and b can be individually* `in`*, but not together.*

2 Overview of results

In [Blümel *et al.*, 2025] we focus on a well-studied fragment of ABA whereby assumptions cannot be inferred (known as *flat* ABA) and where assumptions, their contraries and any sentences in inference rules are atomic. We provide complexity results for deciding independence in both ABA and abstract argumentation frameworks [Rienstra *et al.*, 2020] and propose an SCC-recursive schema for ABA semantics alongside sound polynomial time checks for independence between sets of assumptions in ABAFs. Below, we outline our main contributions.

Independence in ABA We introduce conditional independence to ABA as a means to analyze the relations between the acceptance of disjoint sets of assumptions. To be more precise, we say two sets of assumptions A and B are independent, given a third set C, if, once we know the labellings of the assumptions in C (`in`, `out`, `und`), then the possible labellings for A are no longer influenced by those for B. This notion, captured by Definition 1.4, allows us to identify when reasoning about one part (e.g., A) can be done independently of another (e.g., B). We show our definition behaves as expected and satisfies the semi-graphoid axioms. The complexity of deciding independence, however, turns out to be rather high.

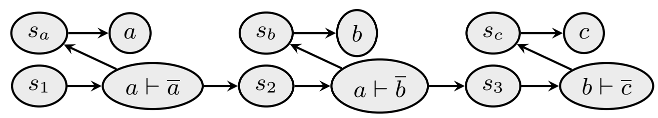
Proposition 2.1. *Let $\mathcal{C}$ denote the class of AFs/flat ABAFs. Deciding σ-independence in $\mathcal{C}$ is Π_2^P-complete for $\sigma \in \{co, st\}$ and Π_3^P-complete for $\sigma = pr$.*

We also show a correspondence to conditional independence for AFs [Rienstra *et al.*, 2020] and settle the complexity of the corresponding decision problems for those.

SCC-recursiveness for ABA To alleviate the high computational complexity we exploit the structure of ABA and explore *SCC-recursiveness* for ABAFs. SCC-recursiveness is well-studied in the realm of abstract argumentation [Baroni *et al.*, 2005; Dvorák *et al.*, 2024]; semantics that satisfy this property can be processed locally, along the *strongly connected components (SCCs)* of a graph. We construct an SCC-recursive scheme for ABAFs around the defeasible rules, which are the core of reasoning in ABA, and provide SCC-recursive characterizations of the standard semantics.

SCC-based Independency Checks We first examine whether we can exploit the close connection between AFs and ABAFs to use the polynomial-time check for independence within AFs proposed by [Rienstra *et al.*, 2020] for ABAFs. The method utilizes the SCC-structure of the induced AF to facilitate a check via d-separation which requires a DAG (directed acyclic graph). For that we use the d-graph of the AF, which has a tree-structure and contains a node for each SCC of the AF, having the contained arguments as children, from which in turn other SCC-nodes are reached. The approach is illustrated in the following example.

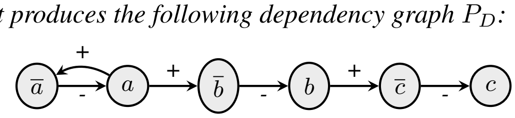
Example 2.2. *Consider an ABAF D with assumptions a, b, c and rules $(\overline{a} \leftarrow a), (\overline{b} \leftarrow a), (\overline{c} \leftarrow b)$. All assumptions of the d-graph G_{F_D} of AF F_D for the ABAF D are in terminal SCCs.*



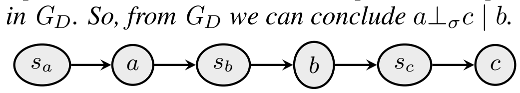
In G_{F_D}, we cannot say if a and c are independent, given b, since they are connected by a collider-free path not containing b.

As the example shows, the AF-SCCs that contain assumption arguments are often either terminal or initial SCCs. For a sensitive independence check we require an SCC-structure which is informative enough. We propose an alternative check using directly the SCC-structure of the ABAF. The SCC-Decomposition of an ABAF is conducted via the dependency graph P_D [Rapberger *et al.*, 2022]. We can now compute the d-graph G_D with respect to said dependency graph and check for independencies using the d-separation criterion. As the example shows, the second approach indeed allows insights on the (in)dependencies between assumptions the AF-based approach cannot procure.

Example 2.3. *Consider again the ABAF D from Example 2.2. It produces the following dependency graph P_D:*



Each assumption has its own SCC in P_D, thus, G_D wrt. P_D is a simple chain. In contrast to Example 2.2, b separates a and c in G_D. So, from G_D we can conclude $a \perp_\sigma c \mid b$.



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